

A STRATEGY FOR THE ANALYSIS OF THE STABILITY OF REFERENCE POINTS IN DEFORMATION SURVEYS

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Confirmation of the stability of reference points is one of the main problems in deformation analysis. The difficulty lies in the datum defects of monitoring networks. A strategy has been developed by the authors and successfully applied in a number of projects. The method leading to the minimization of the first norm of the vector of displacements of reference points has been designed for identifying unstable reference points. Having flagged the unstable reference points, estimation and statistical testing of their displacements are performed.

Two examples are given. A vertical reference network is analyzed step by step to illustrate the proposed strategy. Results of analyzing a horizontal reference network for monitoring a gravity dam are given in the second example.

La confirmation de la stabilité des points de référence constitue l'un des principaux problèmes dans l'analyse des déformations. La difficulté réside dans l'imperfection des données des réseaux de surveillance. Les auteurs ont mis au point une stratégie qu'ils ont appliquée avec succès à un certain nombre de projets. La méthode menant à la minimisation de la première norme du vecteur des déplacements des points de référence a été conçue pour identifier les points de référence instables. Après avoir ainsi identifié ces derniers, une estimation et une analyse statistique de leurs déplacements sont alors effectuées.

Deux exemples sont donnés. Un réseau de référence verticale est analysé de façon systématique pour illustrer la stratégie proposée. Les résultats de l'analyse du réseau de référence horizontale à des fins de surveillance d'un barrage figurent dans le second exemple.

Introduction

Most surveying schemes for monitoring deformations are comprised of several reference points against which the displacements of the object points are calculated. To obtain the absolute displacements of the object points, the stability of the reference points must be ensured and any unstable points identified. Otherwise, the calculated displacements of the object points and the subsequent analysis and interpretation of the deformation of the object may be significantly distorted. Figure 1 illustrates a situation where points A, B, C, and D are reference points and the others, object points. If point B has moved but is not identified and is used with point A as explicit minimal constraints in the adjustment for two campaigns of observations, then all the object points and reference points C and D will show significant movements (even when, in reality, they are stable). The reference points are supposed to be located outside the deformation area. However,

some of them may move due to, for instance, local forces and inappropriate monumentation. Even if the reference points are monumented on solid bedrock, the forces which cause the deformation of the object may also affect the surroundings over a large area. Therefore, the stability of reference points should always be carefully checked. Unfortunately, this problem is very often underestimated and neglected in surveying practice.

Over the past two decades several methods for the analysis of reference networks have been developed in various research centers [Pelzer 1974; van Mierlo 1978; Niemeier 1981; Koch and Fritsch 1981; Chrzanowski et al. 1983; Heck 1983; Janusz 1983; Gründig et al. 1985]. A conceptual review has been given by Chrzanowski and Chen [1986].

One method, developed by the authors, is a special case of the UNB generalized method for

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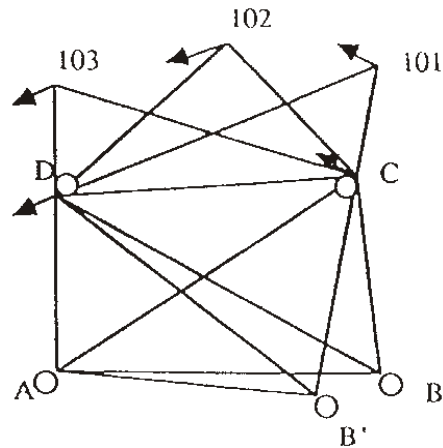


Figure 1: The influence of an unstable reference point (not to scale)

the analysis of deformation surveys. This method has been implemented in the computer package DEFNAN [Chrzanowski et al. 1983; Chen 1983; Secord 1985; Chrzanowski et al. 1986]. Since 1983, the method has been successfully applied to a number of monitoring networks. This paper discusses the basic principles of the method and the analysis strategy.

Adjustment of the Observations in a Free Monitoring Network

Since no reference point in a geodetic monitoring network can be accepted as stable until the analysis is performed, the network must be treated as a free network. It means that the network in itself does not contain enough information to be located in space. Examples are a leveling network without elevation information of any point, or a horizontal trilateration network without the known coordinates of any point and any known azimuth between a pair of points. Therefore, free networks can be freely translated or rotated or scaled in space, and can be considered as suffering from datum defects.

Consider the linearized parametric adjustment model of a free network as

$$l + v = Ax, \quad \text{with } \sigma_o^2 Q \quad (1)$$

where l is the n -vector of observations, v is the n -vector of residuals, x is the vector of the corrections to the approximate coordinates of the survey points, A is the configuration matrix, σ_o^2 is the *a priori* variance factor, and Q is the cofactor matrix of the observations. The least squares criterion leads to the normal equations:

$$Nx = w \quad (2)$$

where $N = A^T Q^{-1} A$, $w = A^T Q^{-1} l$. Due to datum defects in the network, the coefficient matrix, N , of the normal equations is singular, i.e., $\det\{N\} = 0$. Therefore, one must define a datum to solve for x , expressed by a system of constraints or datum equations as

$$D^T x = 0 \quad (3)$$

in which there is an equation for each datum defect of the network. For example, a leveling network of m points with point P_i held fixed, has the datum equation $\delta H_i = 0$ where δH_i is the correction to the approximate height of point P_i , and matrix D^T is of the order 1 by m and has 1 for the i^{th} element and 0 elsewhere; a trilateration network of m points with, say, point P_1 and the azimuth from point P_1 to point P_3 held fixed, then the datum equations are $\delta x_1 = \delta y_1 = 0$ and $\sin(\alpha_{13})\delta y_3 - \cos(\alpha_{13})\delta x_3 = 0$, where δx_i and δy_i are the corrections to the approximate coordinates of point P_i , and matrix D^T (3 by $2m$) is written as

$$D^T = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos(\alpha_{13}) & \sin(\alpha_{13}) & \dots & 0 & 0 \end{bmatrix}$$

The solution of equation (2) with datum equations $D^T x = 0$ reads as [Chen 1983]

$$\hat{x} = (N + DD^T)^{-1} w \quad (4)$$

with a cofactor matrix

$$Q\hat{x} = (N + DD^T)^{-1} \cdot H(H^T DD^T H)^{-1} H^T \quad (5)$$

in which the matrix H fulfills the conditions that $\text{rank}\{H\} = \text{rank}\{D\}$ and $NH = 0$. For a vertical network $H = I$, a vector with all elements equal to 1. For a pure triangulation network of m points,

$$H^T = \begin{bmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 \\ -y_1^0 & x_1^0 & -y_2^0 & x_2^0 & \dots & -y_m^0 & x_m^0 \\ x_1^0 & y_1^0 & x_2^0 & y_2^0 & \dots & -y_m^0 & x_m^0 \end{bmatrix} \quad (6)$$

where x_i^0, y_i^0 are the coordinate components of

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point P_i with respect to the centroid of the network, i.e.,

$$x_i^0 = x_i - (\sum_1^m x_j) / m, \quad y_i^0 = y_i - (\sum_1^m y_j) / m$$

with x_i, y_i the coordinate components of point P_i in the original coordinate system. The rows in H^T correspond to the translations in the x and y directions, rotation, and scale change of the network, respectively. For a trilateration or triangulation network, the last row of H^T in equation 6 is not involved.

In practice, one could introduce some pseudo-observations with very small variances to remove the datum defects. For example, in a trilateration network, the observations of an azimuth and the coordinates of one point are introduced. The results will be practically the same as that obtained from equations (4) and (5). However, one should be cautioned that ill conditioning of the normal equations may occur due to very small variances for the pseudo-observations.

The solution of equation (2) with respect to the datum equations $D^T x = 0$ can also be realized through a similarity transformation from any solution \hat{x}_u as

$$\hat{x} = S\hat{x}_u, \quad Q\hat{x} = SQ_u S^T \quad (7)$$

with

$$S = I - H(D^T H)^{-1} D^T = I - H(H^T W H)^{-1} H^T W, \quad (8)$$

where $W = D(D^T D)^{-1} D^T$. Matrix W in equation (8) can be interpreted as a "weight" matrix in the definition of the datum and therefore equation (7) is called a weighted similarity transformation. If all of the points in the network are of the same importance in defining the datum, then $W = I$ and the results become the "inner constraint" solution. If only some points are used to define the datum, these points are given unit weight and the others zero weight, i.e., $W = \text{diag}\{I, 0\}$. For more details on the adjustment of free networks, reference can be made to Chen [1983].

The *a posteriori* variance factor $\hat{\sigma}_0^2$ and its degrees of freedom, df , are computed from the estimated residuals \hat{v} as:

$$\hat{\sigma}_0^2 = \frac{\hat{v}^T Q^{-1} \hat{v}}{df}, \quad df = n - \text{rank}\{A\} \quad (9)$$

where the rank of A for a complete network (without configuration defects) is equal to the number of unknown parameters x minus the number of datum defects of the network.

Identification of Unstable Reference Points by Minimizing the First Norm of the Displacement Vector of Reference Points

When comparing two campaigns, the vector of displacements for all the surveyed points and its cofactor matrix are calculated as:

$$d = \hat{x}_2 - \hat{x}_1, \quad Q_d = Q_{d1} + Q_{d2} \quad (10)$$

The pooled variance factor $\hat{\sigma}_{0p}^2$ and its degrees of freedom df_p are computable from

$$\hat{\sigma}_{0p}^2 = \{df_1(\hat{\sigma}_{01}^2) + df_2(\hat{\sigma}_{02}^2)\} / df_p, \quad df_p = df_1 + df_2 \quad (11)$$

where the subscripts 1 and 2 refer to the first and second campaigns, if the *a priori* variance factor is not available and the statistical test on the null hypothesis $H_0: \hat{\sigma}_{01}^2 = \hat{\sigma}_{02}^2$, with significance level α ,

$$\{F(\alpha/2; df_2, df_1)\}^{-1} < \hat{\sigma}_{01}^2 / \hat{\sigma}_{02}^2 < F(\alpha/2; df_1, df_2) \quad (12)$$

is true. Failure of the above test may be caused by incompatible weighting of the observations between the two adjustments or by incorrect weighting scheme.

As already mentioned, the displacements calculated from equation (10) may be biased by a pre-selected datum or by different datum definitions in the adjustment of two campaigns, which makes identification of unstable reference points difficult. To overcome this problem a strategy of minimizing the first norm of the displacement vector of the reference points has been developed by the authors [Chen 1983]. The strategy provides a datum which is robust to unstable reference points and gives less distorted displacements. For more on robust estimation, the readers are referred to Huber [1981], and Caspary [1988].

Let d_r and Q_{dr} be the displacement vector and its cofactor matrix for the reference points, respectively, extracted from d and Q_d in equation (10). Transformation of them onto another datum is performed using equations (7) and (8) as

$$\tilde{d}_r = [I - H_r(H_r^T W_r H_r)^{-1} H_r^T W_r] d_r = S_r d_r, \quad (13a)$$

and

$$Q_{\tilde{d}_r} = S_r Q_d S_r^T \quad (13b)$$

Matrix H_r is constructed in the same manner as in the previous section and depends on the union

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of the datum defects in the two campaigns and on the number of reference points. For example, if the monitoring network in the first campaign is triangulation which has datum defects of two translations, one rotation and one scale and in the second campaign, a trilateration which has datum defects of two translations and one rotation, then the union of the datum defects is the same as in the first campaign.

The strategy presented here is to select such a weight matrix W_r in equation (13a) that the first norm of the displacement vector \tilde{d}_r approaches a minimum, i.e., $\|\tilde{d}_r\|_1 = \min$. Let

$$t = (H_r^T W_r H_r)^{-1} H_r^T W_r d_r,$$

called the transformation parameters, then $\|\tilde{d}_r\|_1 = \sum_i |d_r(i) - h_i t|$, where $d_r(i)$ is the i^{th} element of d_r , and h_i the i^{th} row vector of matrix H_r . The condition can be written as

$$\min_t \{ \sum_i |d_r(i) - h_i t| \} \tag{14}$$

Equation (14) may not always have a unique solution. This is, however, not a problem for the purposes of identification of unstable reference points.

For a vertical monitoring network, the datum parameter is a translation quantity t_z in the vertical direction. If w_i is the displacement of point P_i , then expression (14) becomes

$$\min_{t_z} \{ \sum_i |w_i - t_z| \} \tag{15}$$

The solution for t_z is straightforward. All the w_i are arranged in a sequence of their increasing algebraic values, and the middle value is the value t_z . If there is an even number of reference points, either value of the two middle displacements or their average can be used as t_z . In other words, the point or a pair of points whose displacement(s) is in the middle place has weight 1 and the rest, weight 0. The new vector of displacements and its cofactor matrix are calculated from equation (13).

For a two-dimensional network, a method of iterative weighted similarity transformation has been elaborated [Chen 1983; Secord 1985]. In this method, the weight matrix W_r in equation (13) is taken as identity at the outset, then in the $(k+1)^{\text{th}}$ transformation the weight matrix is defined as

$$W_r^{(k+1)} = \text{diag} \{ 1/|\tilde{d}_r^{(k)}(i)| \}, \tag{16}$$

where $\tilde{d}_r^{(k)}(i)$ is the i^{th} component of the vector \tilde{d}_r after the k^{th} iteration. The iterative procedure continues until the absolute differences between the

successive transformed displacement components are smaller than a tolerance δ (say, half of the average accuracy of the displacement components).

During this procedure some $\tilde{d}_r^{(k)}(i)$ may approach zero causing numerical instabilities because $1/|\tilde{d}_r^{(k)}(i)|$ becomes very large. There are two ways to handle this. One is to replace the expression (16) by $W_r^{(k+1)} = \text{diag} \{ 1/(|\tilde{d}_r^{(k)}(i)| + \delta) \}$, and the other

is to set a lower bound. When $|\tilde{d}_r^{(k)}(i)|$ is smaller than the lower bound, its weight is set to zero. If in the following iterations the $\tilde{d}_r^{(k)}(i)$ becomes significantly large again, the weights can be changed accordingly. The explanation for the second way is given in Schlossmacher [1973]. The above procedure provides an approximate solution to equation (14). In the final iteration, say $(k+1)^{\text{th}}$, the cofactor matrix is calculated from:

$$Q\tilde{d}_r = S_r^{(k+1)} Q d_r [S_r^{(k+1)}]^T \tag{17}$$

By comparing the displacement of each point against its confidence region at a specified significance level α , one can identify the reference points which are most probably unstable.

Estimation and Statistical Testing of the Displacements of Unstable Reference Points and Object Points

The final displacements of the points identified as unstable and of all the object points are estimated by a least squares fitting of a deformation model Bc to the displacements d obtained from equation (10) as

$$d + v = Bc \tag{18}$$

where v is the vector of residuals after fitting, c is the vector of the final displacements to be estimated and B is the design matrix. Explicitly, the deformation model for each unstable point and object point P_j in a two dimensional network is written as:

$$d_j + v_j = \begin{bmatrix} a_i \\ b_i \end{bmatrix} = c_i \tag{19a}$$

and for each stable point P_j as:

$$d_j + v_j = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19b}$$

Thus, the matrix B in equation (18) has unit elements corresponding to the unstable points and

For a two-dimensional network, a method of iterative weighted similarity transformation has been elaborated . . .

object points, and zeros elsewhere. Solution of equation (18) gives

$$\hat{c} = (B^T P_d B)^{-1} B^T P_d d \quad (20a)$$

and its cofactor matrix

$$Q\hat{c} = (B^T P_d B)^{-1} \quad (20b)$$

The weight matrix P_d can be calculated [Chen, 1983] either as

$$P_d = N_1(N_1+N_2)^{-1}N_2 \quad (21)$$

or

$$P_d = (SQ_d S)^+ = [SQ_d S + H(H^T H)^{-1} H^T]^{-1} H(H^T H)^{-1} H^T \quad (22)$$

In equation (21) N_i ($i=1,2$) is the coefficient matrix of the normal equations (see equation (3)). A generalized inverse, $(N_1+N_2)^+$, can be computed as $(N_1+N_2+HH^T)^{-1}$, where the column vectors of H correspond to the *common datum defects* in the two campaigns. If two campaigns have the same survey scheme and measurement accuracy, i.e., $N_1=N_2=N$, then

$$P_d = N/2. \quad (21')$$

In equation (22) matrix S is as expressed in equation (8) with $W=I$ and the column vectors of H correspond to the *union of datum defects* in the two campaigns. The reason for computing the weight matrix in such a way is so that the estimated parameters \hat{c} will be independent of the datum used in the adjustments. If the datum defects are removed by the introduction of some pseudo-observations with small variances, then the weight matrix could be calculated from

$$P_d = Q_d^{-1} \quad (22')$$

However, in this case, not only will numerical problems likely occur due to ill-conditioning of Q_d but also will complications arise in modelling of deformations. Some additional parameters have to be introduced, as is explained in the second example below. More details are given in [Chrzanowski et al. 1983].

The significance of the estimated displacement \hat{c}_i for an unstable point P_i is indicated by

$$\hat{c}_i^T Q\hat{c}_i^{-1} \hat{c}_i / [m_{ci}(\hat{\sigma}_{0p}^2)] > F(\alpha; m_{ci}, df_p) \quad (23)$$

where m_{ci} is the dimension of \hat{c}_i , $Q\hat{c}_i^{-1}$ is the sub-matrix of $Q\hat{c}$, and $\hat{\sigma}_{0p}^2$ and df_p are the pooled variance factor and its degrees of freedom, respectively. To test the null hypothesis that no other unstable point exists, a quadratic function ΔR of the estimated residuals \hat{v} is calculated as

$$\Delta R = \hat{v}^T P_d \hat{v} \quad (24)$$

which follows a chi-squared distribution with degrees of freedom as

$$df_c = \text{rank}(P_d) - m_c \quad (25)$$

where m_c is the dimension of unknown vector c and the rank defect of P_d is equal to the number in the union of datum defects in both campaigns. If the inequality

$$\Delta R / (df_c \hat{\sigma}_{0p}^2) < F(\alpha; df_c, df_p) \quad (26)$$

holds, the null hypothesis is acceptable at the $(1-\alpha)\%$ confidence level. Otherwise a search for other unstable reference points should be made. The latter case seldom occurs. When the *a priori* variance factor σ_0^2 is known, $\hat{\sigma}_{0p}^2$ and df_p in the tests (23) and (26) are replaced by σ_0^2 and ∞ , respectively.

The analysis procedures discussed above are summarized in Figure 2.

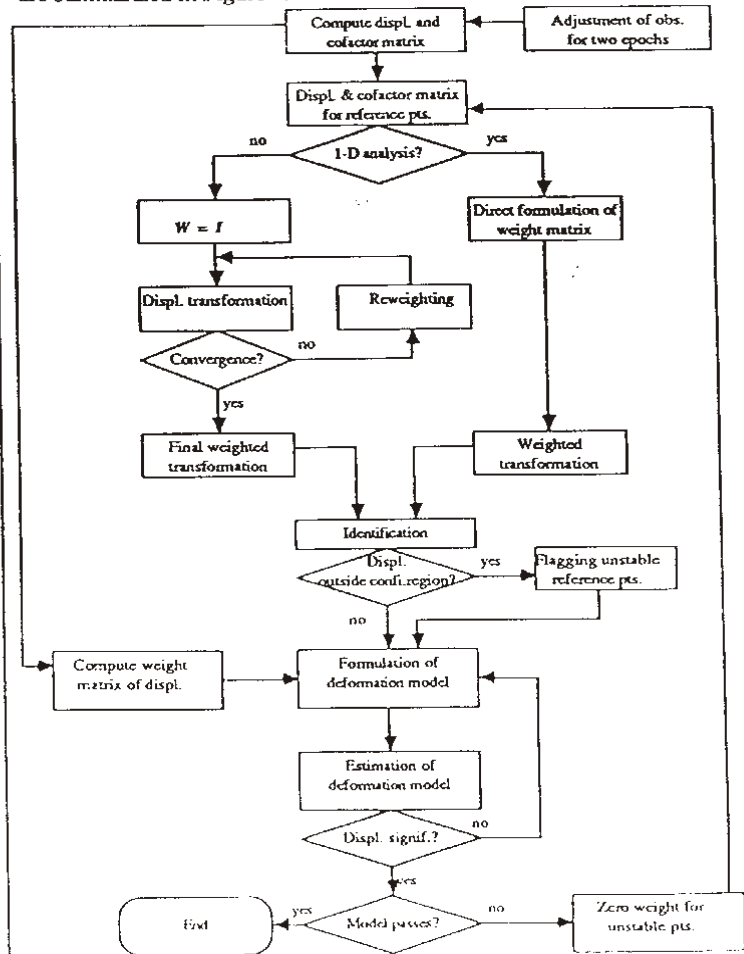


Figure 2: The flowchart of analysis procedures

Examples

Analysis of a Vertical Reference Network

Figure 3 is a leveling reference network with two survey campaigns. The observations are listed in Table 1.

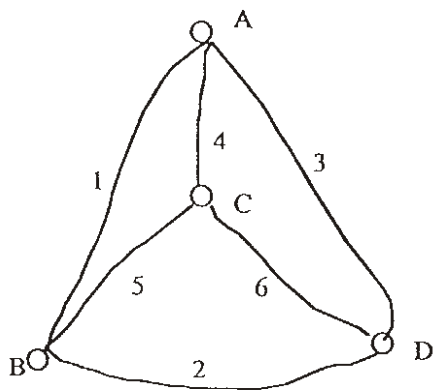


Figure 3: A leveling network

Table 1: The observations of the leveling network.

leveling line	observed height difference [mm]		weight p_i
	campaign 1	campaign 2	
1	45.2	46.9	1
2	265.8	265.6	2
3	310.3	312.2	1
4	-26.2	-24.1	2
5	70.8	70.7	2
6	336.5	336.1	2

Step 1. Adjustment of the leveling network and computation of the displacements

Point A is fixed with an elevation of 0.50000 m in the adjustment of the observations for each campaign. The adjusted heights [m] of points A, B, C, and D for both campaigns are

$$\hat{x}_1^T = (0.50000 \ 0.54479 \ 0.47392 \ 0.81046)$$

and

$$\hat{x}_2^T = (0.50000 \ 0.54673 \ 0.47598 \ 0.81220)$$

Since the *a priori* variance factor is not available, the *a posteriori* variance factors for both campaigns are estimated from the residuals as

$$\hat{\sigma}_{01}^2 = (\sum_1^6 P_i \hat{v}_i^2) / df_1 = 0.269 / 3 = 0.0897$$

and

$$\hat{\sigma}_{02}^2 = (\sum_1^6 P_i \hat{v}_i^2) / df_2 = 0.109 / 3 = 0.0363$$

The null hypothesis $H_0: \hat{\sigma}_{01}^2 = \hat{\sigma}_{02}^2$ is tested using expression (12):

$$1/F(0.025; 3, 3) < \hat{\sigma}_{01}^2 / \hat{\sigma}_{02}^2 = 12.47 < F(0.025; 3, 3) = 15.4$$

Therefore, the pooled variance factor is calculated from equation (11) as

$$\hat{\sigma}_{0p}^2 = 0.0630$$

with degrees of freedom $df_p = 6$.

The vector of displacements [mm] and its cofactor matrix read

$$d = \hat{x}_2 - \hat{x}_1 = (0.00 \ 1.94 \ 2.06 \ 1.74)^T$$

and

$$Qd = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.74 & 0.40 & 0.45 \\ 0 & 0.40 & 0.60 & 0.40 \\ 0 & 0.45 & 0.40 & 0.74 \end{bmatrix}$$

Step 2. Identification of unstable points

Using the method discussed above, the displacements are arranged in the sequence of their increasing algebraic values as (0.00 1.74 1.94 2.06). Thus points D and B are assigned unit weight and points A and C zero weight (translation parameter t_z in equation (15) is the mean of the two middle displacements, i.e., $t_z = 1.84$). After the weighted similarity transformation, the new vector of displacements [mm] and its cofactor matrix are

$$\tilde{d}^T = (-1.84 \ 0.10 \ 0.22 \ -0.10)$$

and

$$\tilde{Q}d = \begin{bmatrix} 0.60 & 0.00 & 0.20 & 0.00 \\ 0.00 & 0.15 & 0.00 & -0.15 \\ 0.20 & 0.00 & 0.40 & 0.00 \\ 0.00 & -0.15 & 0.00 & 0.15 \end{bmatrix}$$

The displacement of each point is tested at a significance level of $\alpha = 0.05$, i.e.,

$$\hat{\sigma}_{q11}^2 = (-1.84)^2 / [(0.60)(0.0630)] = 89.6 > F(0.05; 1, 6) = 6.0$$

$$d_{B1}^2 / (\hat{\sigma}_{q12}^2 + d_{C1}^2 / \hat{\sigma}_{q44}^2) = 10.6 < F(0.05; 1, 6) = 6.0$$

$$d_{C1}^2 / \hat{\sigma}_{q33}^2 = 1.92 < F(0.05; 1, 6) = 6.0$$

It is clear that only point A can be strongly suspected as being unstable.

Step 3. Estimation of the displacement of the unstable point.

Using equation (18) with $d^T = (0.00 \ 1.91 \ 2.06 \ 1.74)$, $B^T = (1 \ 0 \ 0 \ 0)$ and

$$P_d = N_1(N_1 + N_2)N_2 = N/2 = \begin{pmatrix} 2 & -0.5 & -1 & -0.5 \\ -0.5 & 2.5 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -0.5 & -1 & -1 & 2.5 \end{pmatrix}$$

the estimated displacement of point A is

$$\hat{c} = -1.95 \text{ mm}$$

and its cofactor

$$q_c = (B^T P_d B)^{-1} = 0.5$$

The displacement is significant due to the fact that

$$\hat{c}^2 / (\hat{\sigma}_c^2 q_c) = 126.1 > F(0.05; 1, 6) = 6.0$$

The test on the deformation model is performed using the quadratic function (24) with the residuals equal to

$$\hat{v}^T = (-1.95 \ -1.94 \ -2.06 \ -1.74)$$

and

$$\Delta R = \hat{v}^T P_d \hat{v} = 0.189$$

The test

$$\Delta R / (2\hat{\sigma}_q^2) = 1.5 < F(0.05; 2, 6) = 5.1$$

indicates that the deformation model is acceptable, i.e., the remaining points can be considered as stable at 95% confidence level.

One can also use the vector of displacements \hat{d} after the weighted similarity transformation to estimate \hat{c} and calculate the test statistic ΔR . The results will be identical. This indicates that whatever minimal constraint solution is used in the estimation and test processes, it will not affect the final results.

Analysis of the Reference Network for Monitoring a Gravity Dam

A pure triangulation network of 6 reference points and 10 uniquely intersected points on a

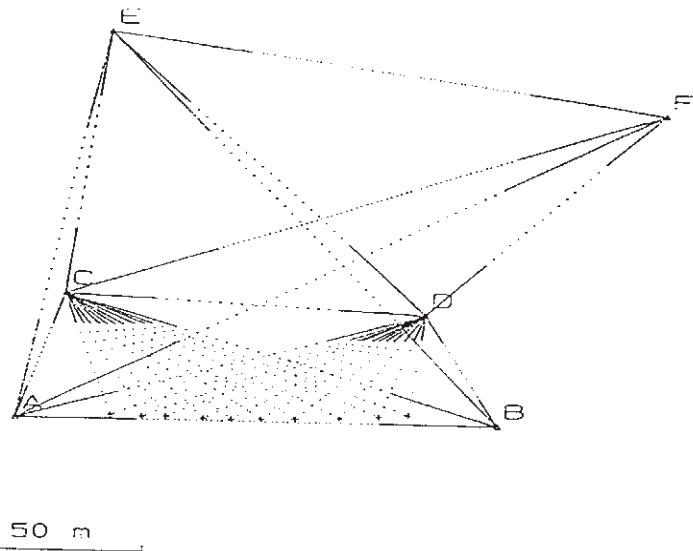


Figure 4: The monitoring network of a gravity dam

dam crest (Figure 4) was observed in two survey campaigns with 47 directions in the first campaign and 53 directions in the second. Least squares estimations of the coordinates \hat{x}_1, \hat{x}_2 were made under explicit minimal constraints involving points E and F (considered "fixed" and errorless). No observation in either campaign was detected as being an outlier at $\alpha=0.05$ using τ -max test [Pope 1976; Vaníček and Krakiwsky 1982]. The pooled variance factor, $\hat{\sigma}_p^2 = 0.95$, had $df_p = 31$ degrees of freedom. The vector of displacements d and the cofactor matrix Q_d were obtained using equation (10). Within the d and Q_d there are zero elements corresponding to points E and F.

The vector of displacements d_r and its cofactor matrix Q_{dr} for the reference points A, B, C, D, E, and F were extracted from d and Q_d . The iterative weighted transformation resulted in the displacement pattern, coupled with the 95% confidence ellipses, for the reference points as shown in Figure 5. Obviously, reference point D has moved significantly while the others remain stable.

Having a_i and b_i as unknown parameters corresponding to the x - and y -components of the displacement for each object point and also for point D, the deformation model consisted of 22 parameters (i.e., 11 pairs of displacement components). The weight matrix P_d was calculated using equation (22) and the 22 unknown parameters were estimated using equation (20). A plot of these estimated displacements and their asso-

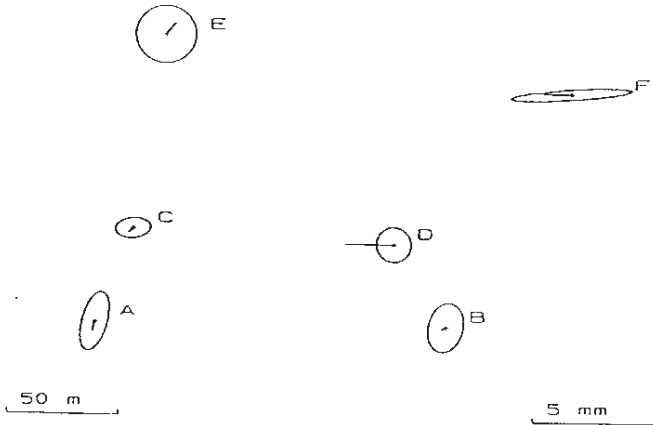


Figure 5: Displacements after the iterative weighted transformation

ciated 95% confidence ellipses is given in Figure 6. With 6 degrees of freedom in the modeling, the test on the adequacy of the model was not rejected at $\alpha=0.05$ since:

$$\Delta R / (df \cdot \hat{\sigma}_p^2) = 1.76 < F(0.05; 6, 31) = 2.41$$

where ΔR is the quadratic form of the residuals \hat{v} after fitting, calculated from equation (24). Thus, the deformation model, in which the reference points A, B, C, E, and F are accepted as stable, is appropriate.

One could perform the above analysis using a subvector d_1 of d and submatrix Q_{d1} of Q_d , which do not include zero elements corresponding to the "fixed" points E and F. In this case, the weight matrix of d_1 would be calculated directly as $P_{d1} = Q_{d1}^{-1}$ rather than using equation (22). Since points E and F have been identified as stable, the deformation model would be formulated in the same way as the above, i.e., using equations (19a) and (19b) except that points E and F would not be involved in modeling. The same estimation and

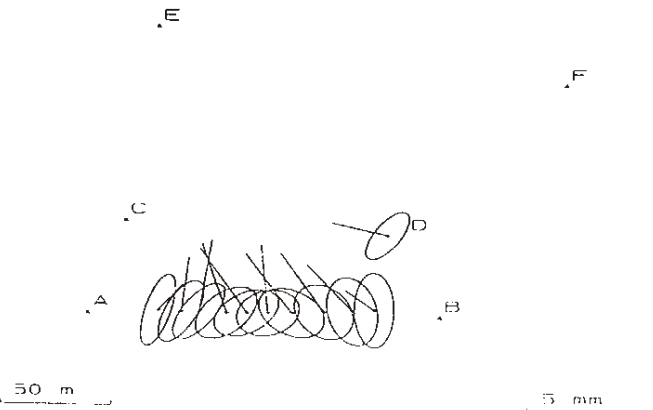


Figure 6: Final displacements of the unstable reference point and the object points

testing procedures would be used. However, if either or both of points E and F would have been identified as unstable, additional parameters which are a function of the displacements of points E and F would have to be included in deformation modeling. If both points would have been identified as unstable reference points, the calculated displacements of other points would have been distorted by translation, rotation and scale change. To account for these effects equations (19a) and (19b) would have to be changed to

$$d_i + v_i = \begin{bmatrix} a_i + k_1(x_i - x_F) + k_2(y_i - y_F) + a_F \\ b_i - k_2(x_i - x_F) + k_1(y_i - y_F) + b_F \end{bmatrix}$$

and

$$d_j + v_j = \begin{bmatrix} k_1(x_j - x_F) + k_2(y_j - y_F) + a_F \\ k_2(x_j - x_F) + k_1(y_j - y_F) + b_F \end{bmatrix}$$

respectively, where k_1 and k_2 are the unknown scale change and rotation parameters. The final displacement components of point E can be then calculated from

$$\hat{a}_E = \hat{k}_1(x_E - x_F) + \hat{k}_2(x_E - x_F) + \hat{a}_F, \quad \hat{b}_E = \hat{k}_1(y_E - y_F) - \hat{k}_2(x_E - x_F) + \hat{b}_F$$

If only one of them, say point E, would have been identified as unstable, a_F and b_F would disappear in the above formulation. To avoid these problems the methodology suggested in this paper should be followed.

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